**Linear Path Skylines in Multicriteria Networks**

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**Multicriteria Routing**
- Find routes between two nodes in a graph.
- Multiple costs per graph edge:
  - e.g. estimated travel distance, estimated travel time, crossings, energy loss, hazard score, discomfort score, ...
- Find routes minimising several cost criteria at once and consider multiple results in case of conflicting costs.

**Introduction to Linear Skylines**

Let $\mathcal{W}^d = \{ w \mid w \in \mathbb{R}^d, \forall 0 \leq i < d, w_i \geq 0, \exists \ w_i > 0 \}$

**Linear Skyline $\preceq$ Skyline**
Comparison for 3 cost objectives, view from the origin:

- **Path Skyline**
- **Linear Path Skyline**

**Contribution 1: Linear Skyline Convex Hull**
Removing the undesired parts from convex hull: $\text{INF} = \{[\infty,0,...,0], [0,\infty,...,0],..., [0,0,...,\infty] \}$

**Contribution 2: $\varepsilon$-Linear Path Skyline**
- Subset of Linear Path Skyline
- Contains for each $w \in \mathcal{W}^d$ an element $y$, such that $w^T y \leq (1+\varepsilon)(\min_{x \in \mathcal{X}} w^T x)$.
- Requires only slight modification to incremental computation of Linear Path Skyline.

**Experimental Evaluation**

*Experimental Setup:* 702 routing tasks on Open Street Map graph around Munich and routing tasks between corners of $n \times n \times n$ lattice graphs for different $n$.

*Linear Path Skyline:* proposed approach outperforms existing ones for higher problem complexities.

*$\varepsilon$-Linear Path Skylines:* as intended, computation time and result size negatively correlated with $\varepsilon$. 

**Contributions (cont’d):**
- Incremental Computation
- Completeness
- Efficiency
- Applications